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Slide of the Seminar

The spatiotemporal spectrum of turbulent flows

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The spatiotemporal spectrum of turbulent flows

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Roma, Italia
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Turbulence under the effect of waves

Start with energy equation in Fourier space

$$\frac{1}{2}\frac{\partial u_k^2}{\partial t} = -i \sum_{k=p+q} u_k^* \cdot (u_q \cdot q) u_p$$

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$\Rightarrow \omega_k = \omega_p + \omega_q$ to have interaction!

Waves are known to...

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- Change the very nature of nonlinear interaction *Nazarenko (2011)*

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- Space and time resolved spectra (e.g. *Yarom and Sharom, Nature Physics (2014)* and *Cobelli et al, PRL (2009)*) can study the effect of waves directly
- Results for rotating, stratified and quantum turbulence.

Simulations

- GHOST: Parallel pseudospectral code with periodic boundary conditions (Gomez et al 2005, Mininni et al 2011)
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Rotating turbulence
Navier Stokes in a rotating frame

$$\frac{\partial \mathbf{u}}{\partial t} = (\nabla \times \mathbf{u}) \times \mathbf{u} - \overbrace{2\Omega \times \mathbf{u}}^{\text{Coriolis}} - \underbrace{\nabla p}_{\text{total pressure}} + \nu \nabla^2 \mathbf{u} + \overbrace{\mathbf{F}}^{\text{forcing}}$$

Rotation axis is along $\hat{z}$ (parallel direction)

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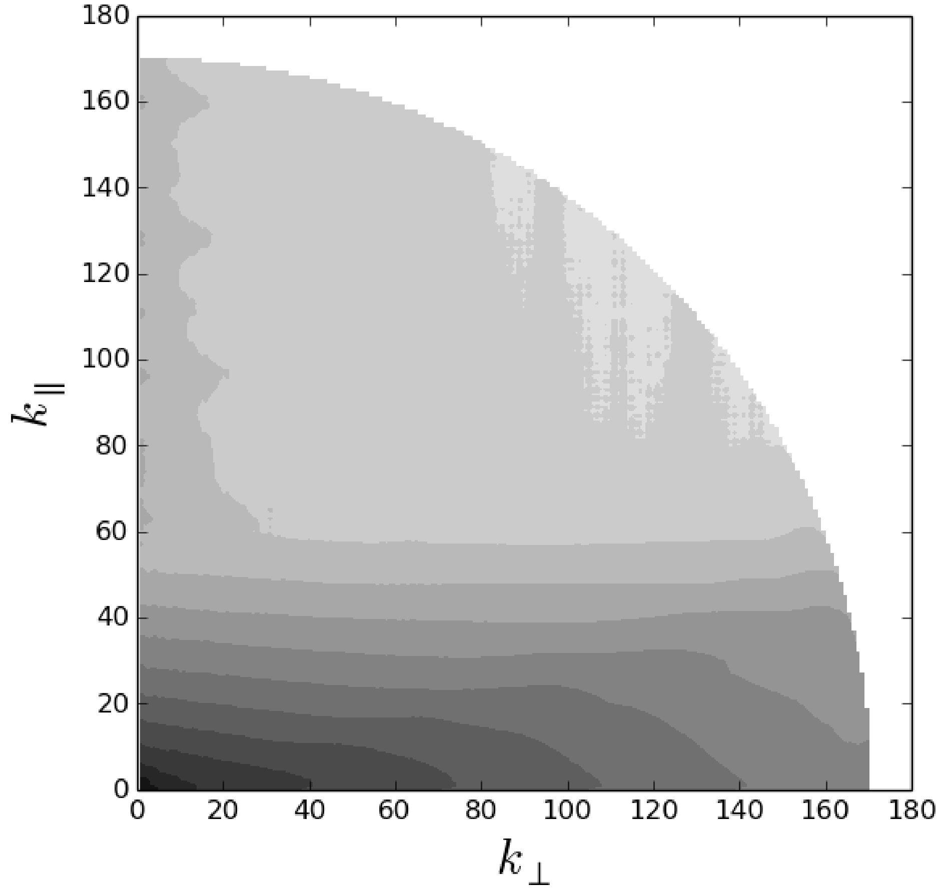
Inertial waves: $\omega_R = \frac{2\Omega k_{||}}{k} \Rightarrow$ Preferential energy transfer towards modes with small $k_{||}$ (Waleffe, PoF 93)

Rotating turbulence

$e(k_\perp, k_\parallel)$

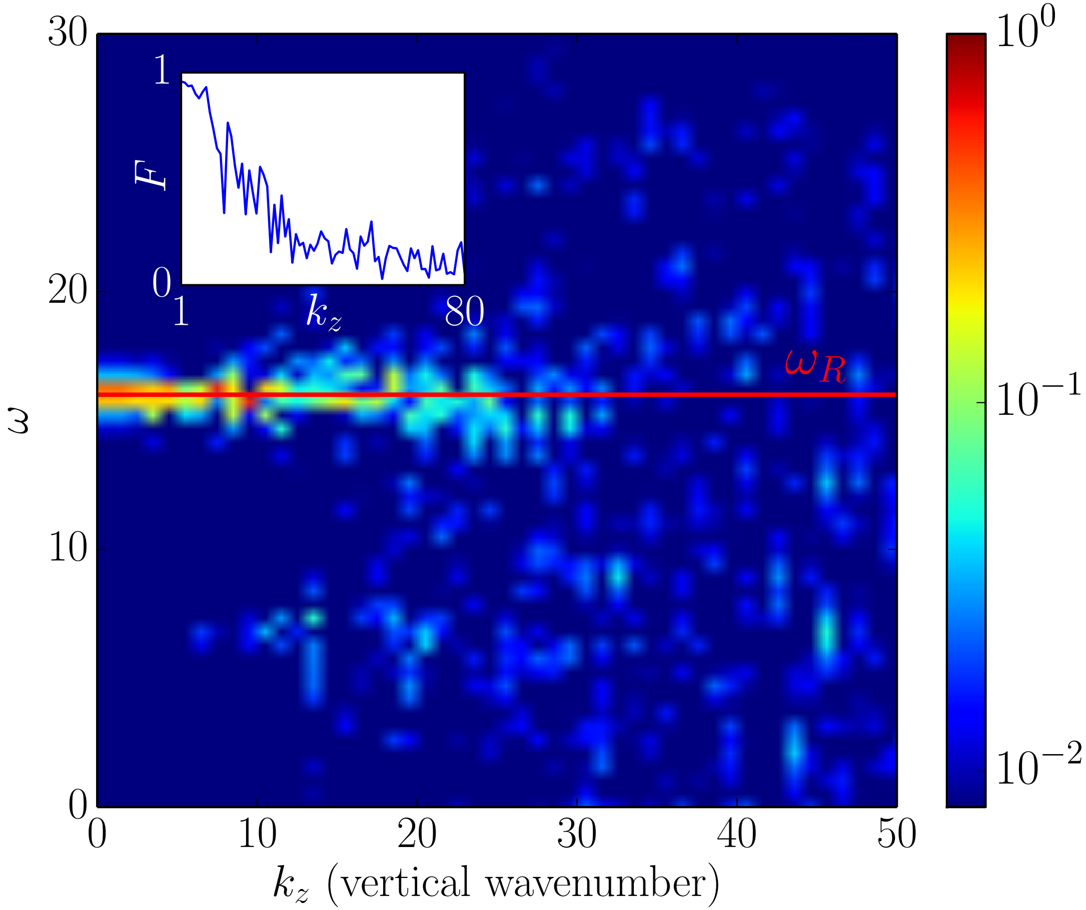


Waleffe's prediction holds! But exactly where are the waves?
Clark di Leoni et al, PoF (2014)

Rotating turbulence

$E(k, \omega)$

Only in the larger scales energy accumulates along modes satisfying the dispersion relation of inertial waves!

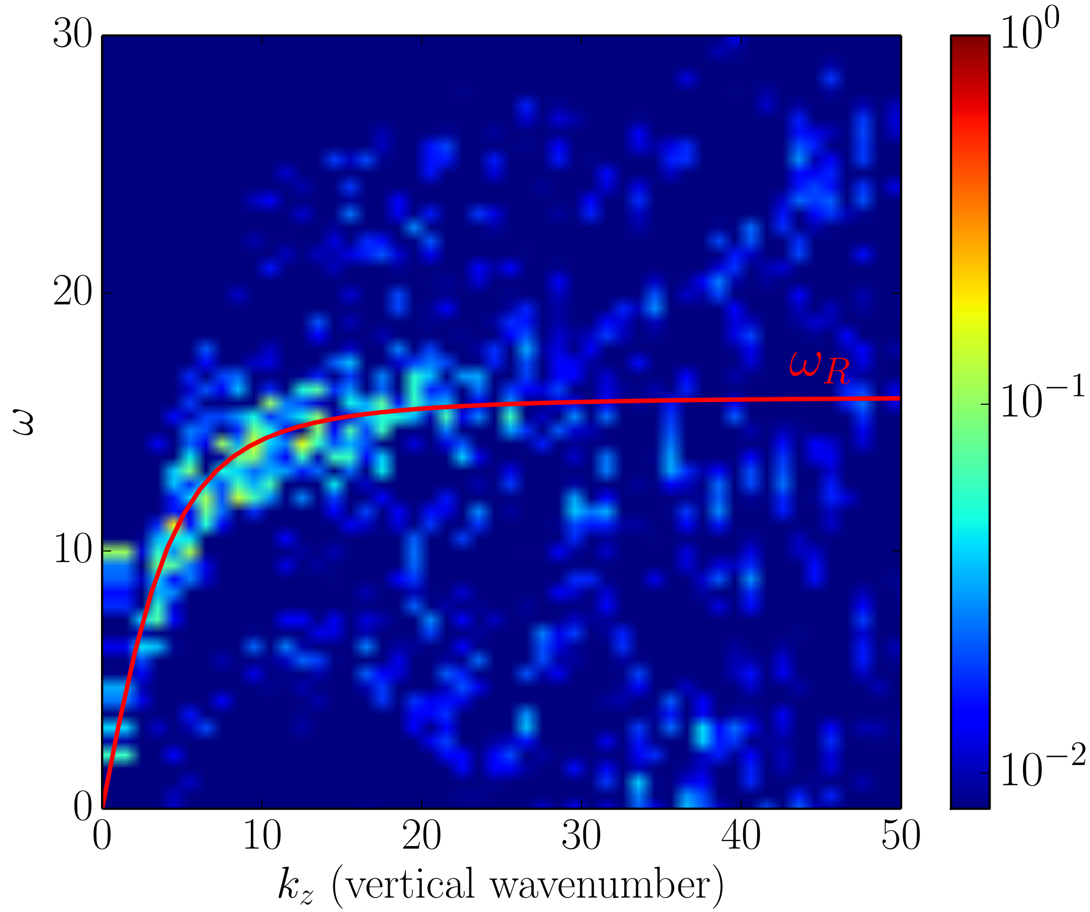


$$\omega_R(0, 0, k_z) = 2\Omega$$

Rotating turbulence

$E(k, \omega)$

"Loss" of waves is not due to isotropization, but because sweeping mechanisms become faster at those scales



$$\omega_R(0,1,k_z) = \frac{2\Omega k_z}{\sqrt{1+k_z^2}}$$

Stratified turbulence
Boussinesq model with no rotation

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \overbrace{N\theta\hat{z}}^{\text{buoyancy}} - \nabla p + \nu\nabla^2\mathbf{u} + \overbrace{\mathbf{F}}^{\text{forcing}}$$
$$\frac{\partial \theta}{\partial t} = \mathbf{u}\cdot\nabla\theta - N u_z - \kappa\nabla^2\theta$$

Stratification gradient is along $\hat{z}$ (parallel direction)

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Internal waves: $\omega_S = \frac{Nk_\perp}{k} \Rightarrow$ Preferential energy transfer towards modes with small $k_\perp$ Waves now travel in the same direction as the mean flow

Stratified flows
Previous works

- Develop vertically sheared horizontal winds *Smith & Waleffe, JFM (2003)*

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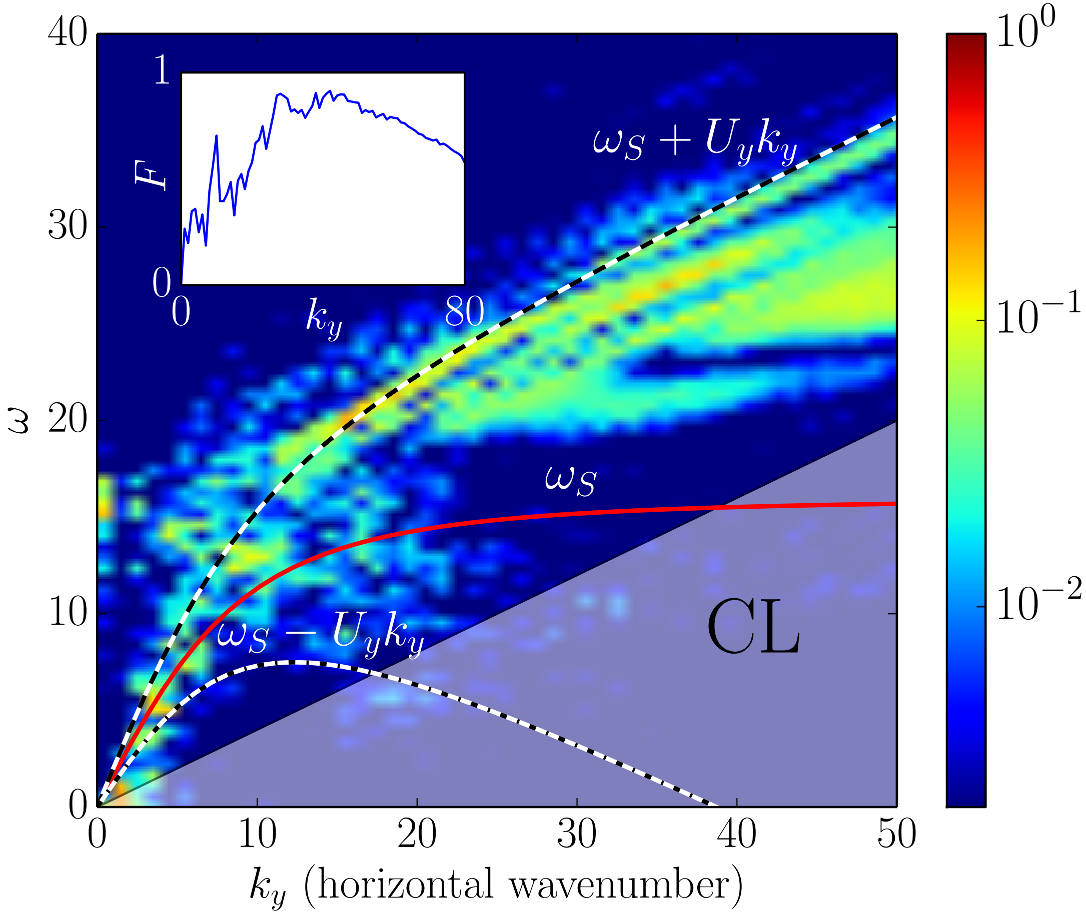
- Theories of stratified turbulence don't take this effects into account!

Stratified turbulence

$E(k, \omega)$



Stratified turbulence

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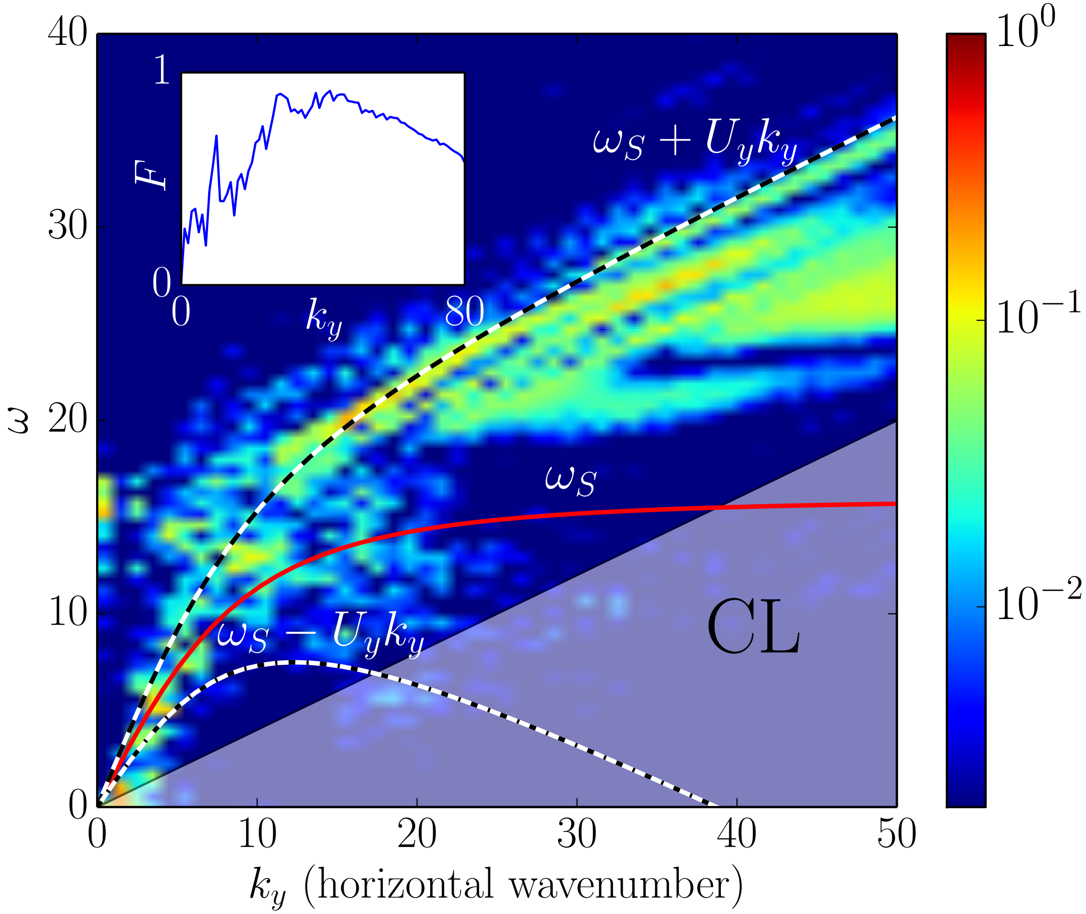

Doppler shifting and Critical Layer absorption appear! This indicates a nonlocal transfer of energy from the small to the large scales.
Clark di Leoni and Mininni, PRE (2015)

Superfluid turbulence
Gross-Pitaevskii equation

Nonlinear PDE describing a Bose Einstein condensate for wavefunction

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi$$

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$$\psi(\mathbf{r},t) = \sqrt{\frac{\rho(\mathbf{r},t)}{m}}e^{i\frac{m}{\hbar}\phi(\mathbf{r},t)}$$

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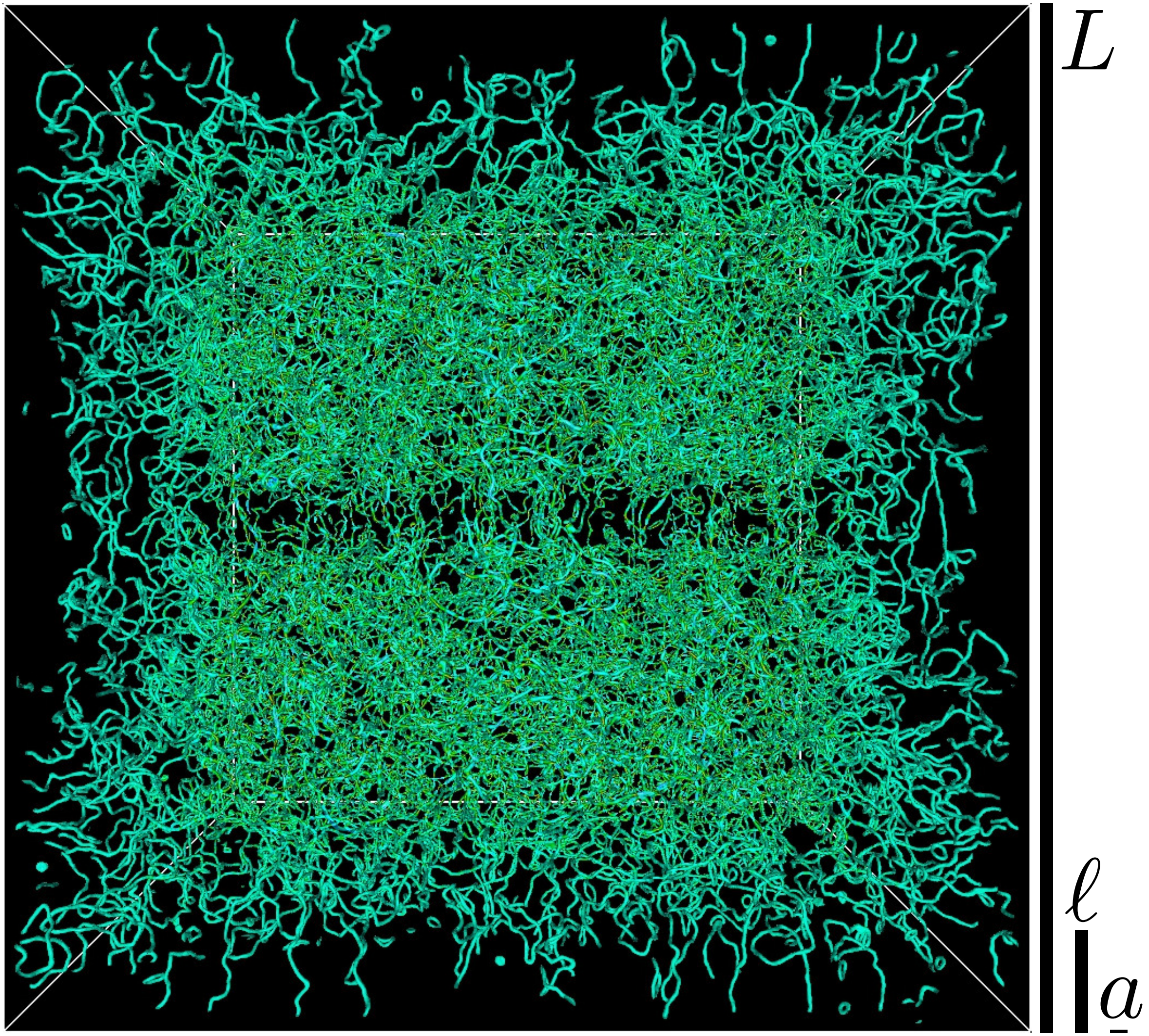
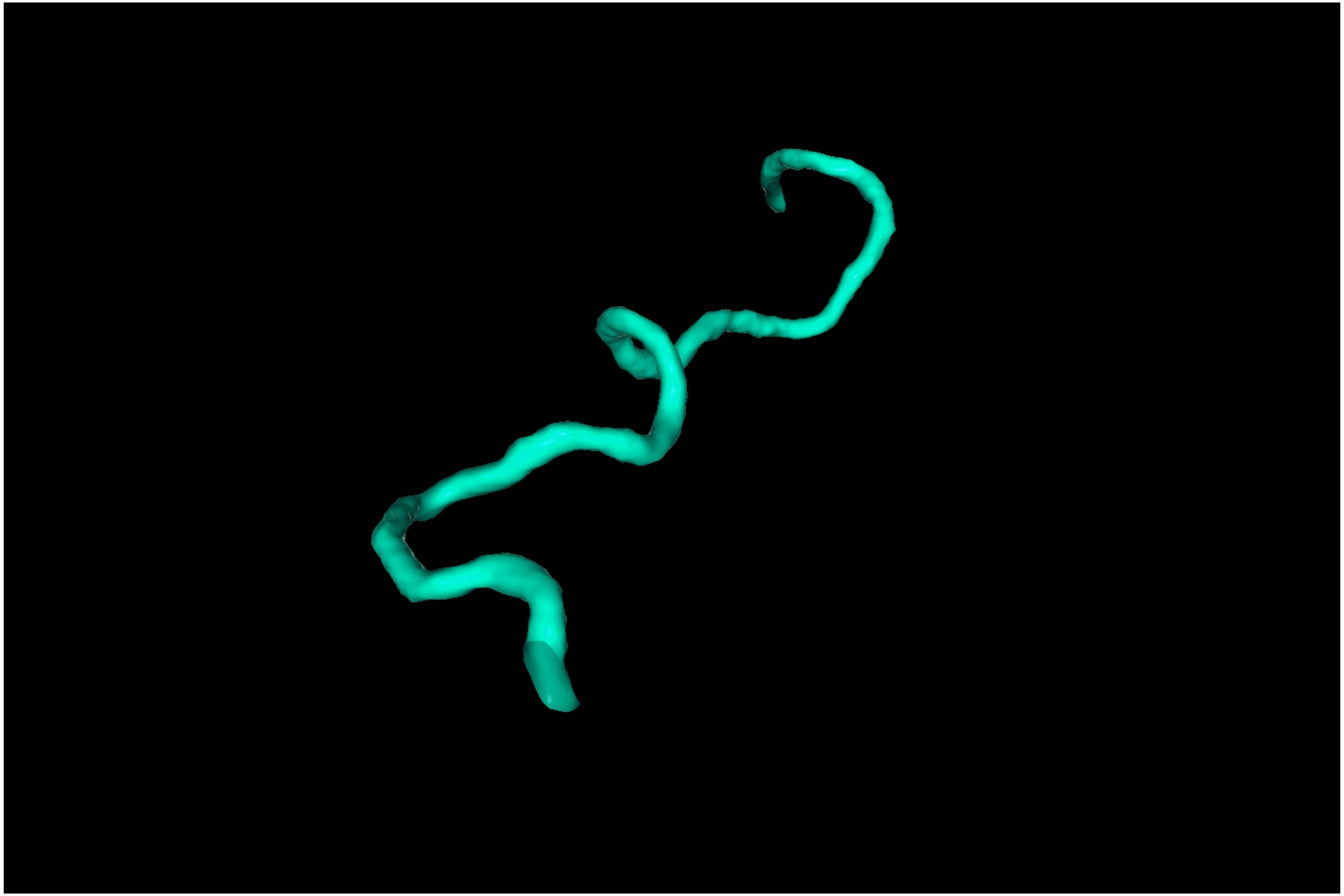
Vorticity is quantized and concentrated along lines with $\rho = 0$

Superfluid turbulence (Gross-Pitaevskii equation)

$\rho(\mathbf{r})$



Kelvin waves

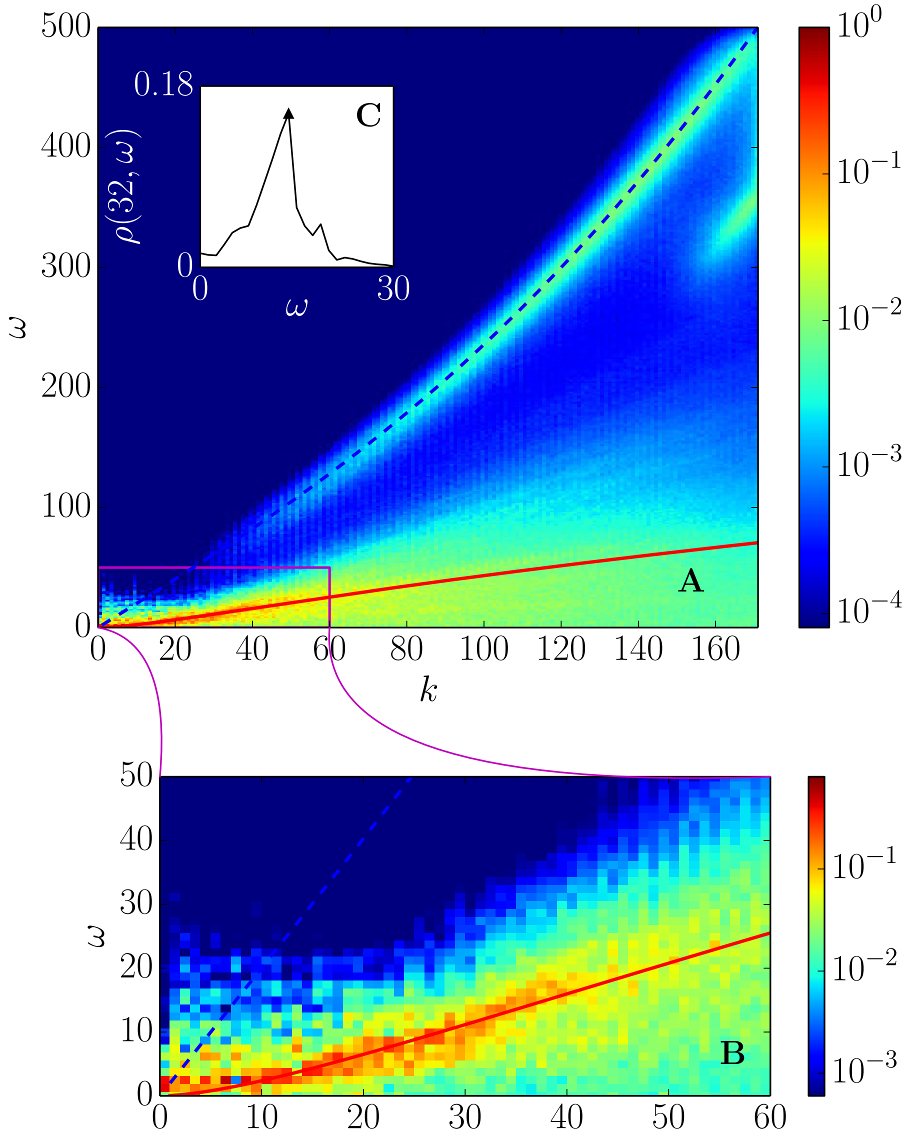


Vortex linex have tension and Kelvin waves can travel through them.
Below the inervortex scale we can have Kelvin wave turbulence.
Sound waves are also present

Superfluid turbulence (Gross-Pitaevskii equation)

$\rho(k, \omega)$

Sound and kelvin waves!

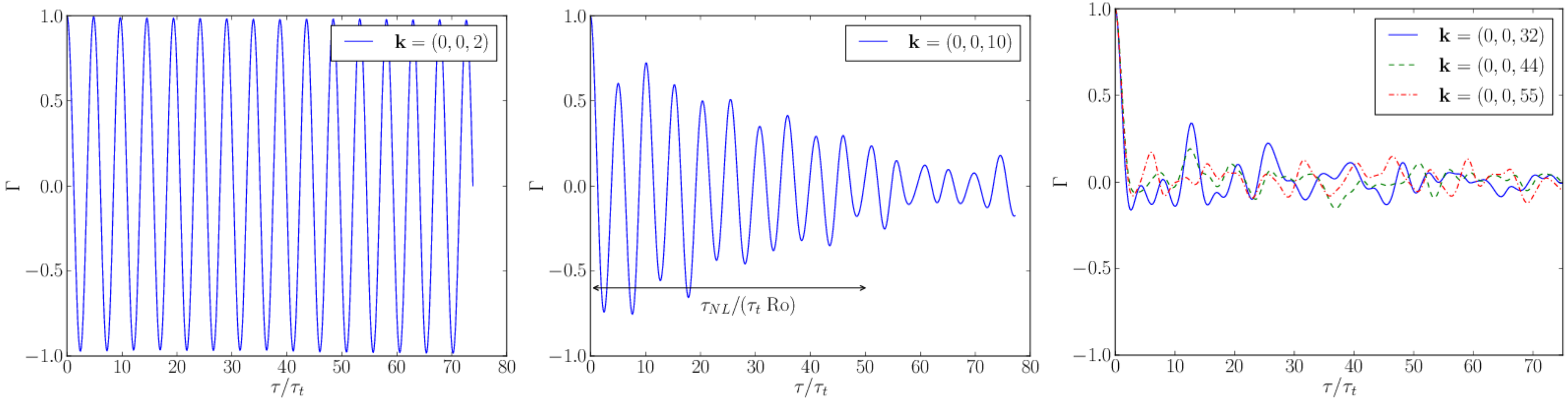


Thanks!

Rotating turbulence
Time correlation functions

We studied the time correlation functions for different modes

$$\Gamma_{ij}(\mathbf{k},\tau) = \frac{\langle \hat{u}_i^*(\mathbf{k},t)\,\hat{u}_j(\mathbf{k},t+\tau)\rangle_t}{\langle |\hat{u}_i^*(\mathbf{k},t)\,\hat{u}_j(\mathbf{k},t)|\rangle_t}$$

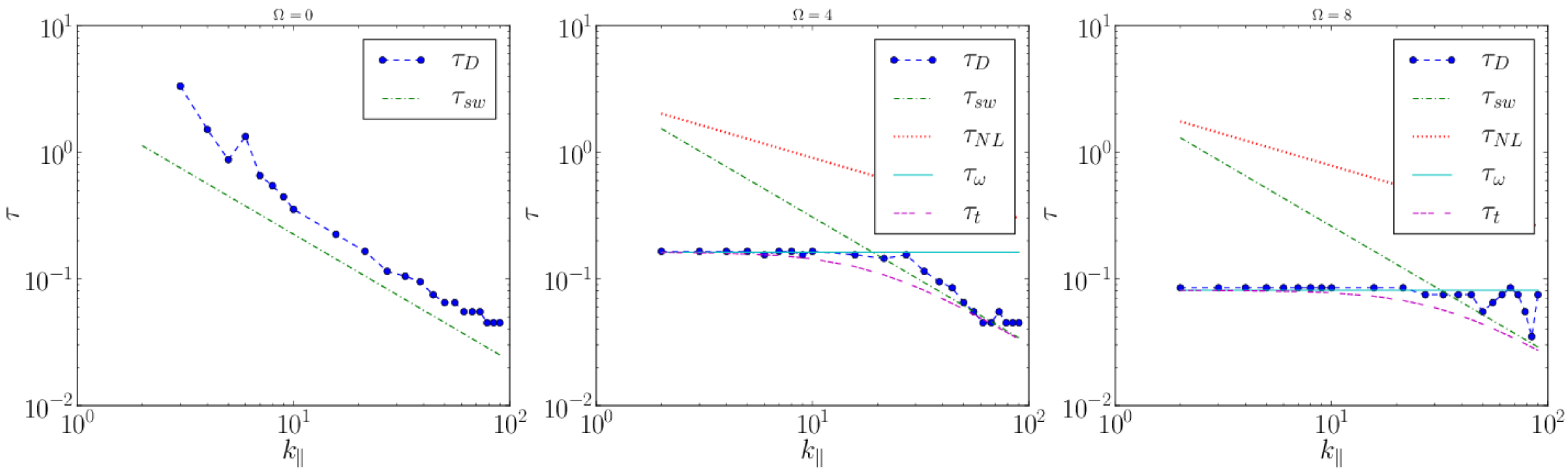


All modes with just parallel components.

Rotating flow

Behaviour of the decorrelation time

τ_{sw}: sweeping time; τ_{NL}: nonlinear time; τ_ω: wave period



$$\tau_t = \left(\tau_{sw}^{-2} + \tau_\omega^{-2}\right)^{-1/2}$$

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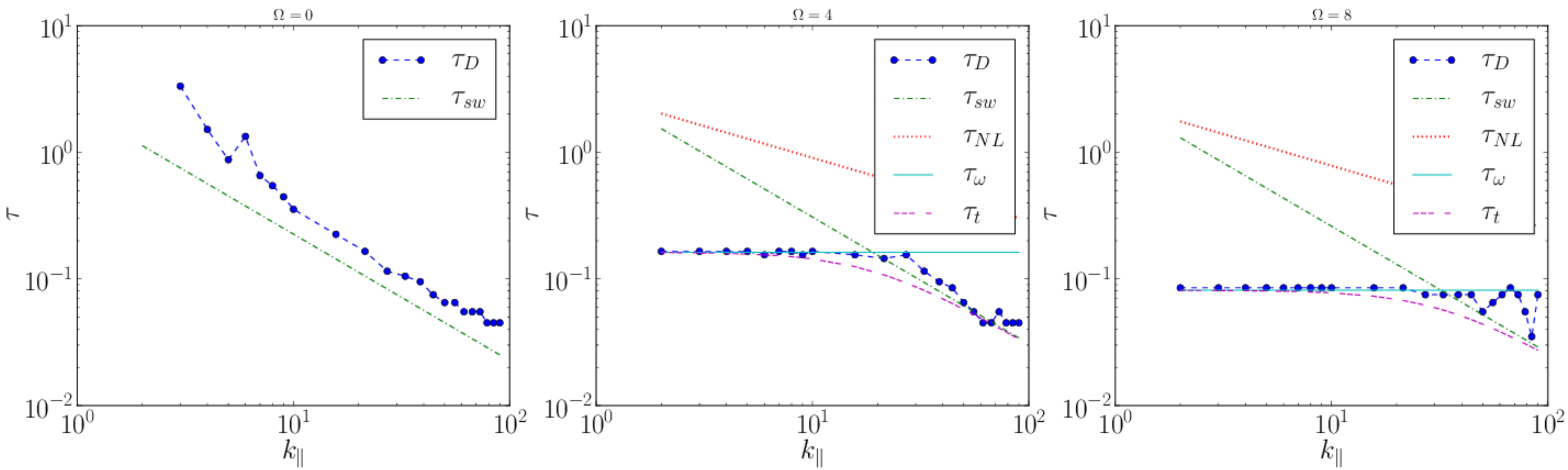

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Interaction is carried out by the fasted mechanism!
For the isotropic case decorrelation is governed by sweeping effects as predicted by Chen & Kraichnan 1989.

Rotating flow

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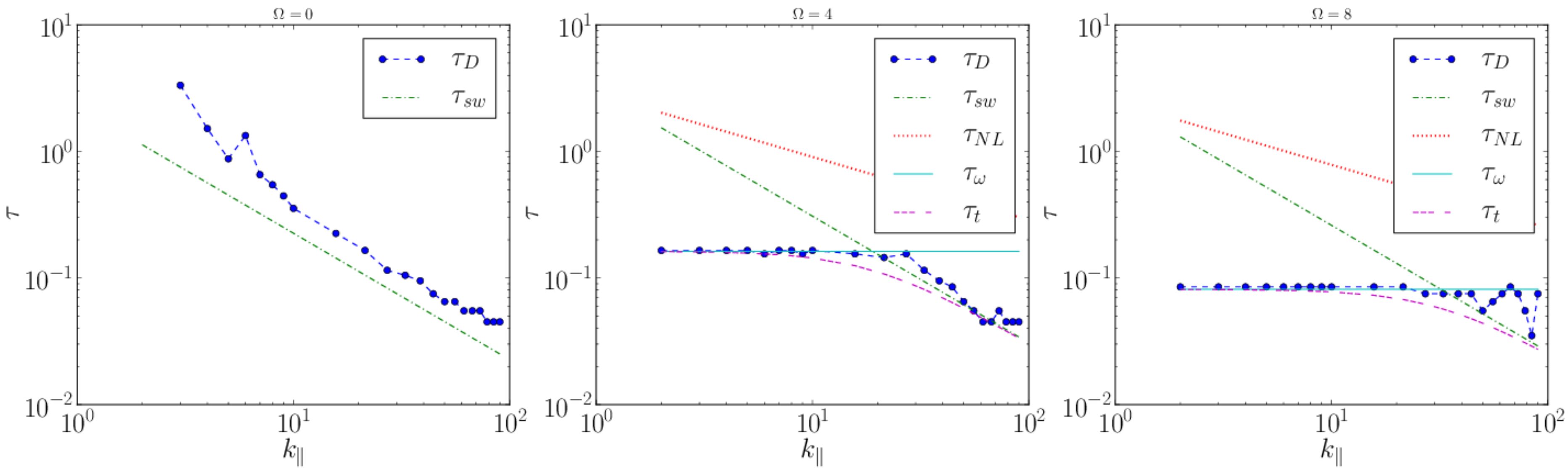

$$\tau_t = \left(\tau_{sw}^{-2} + \tau_\omega^{-2}\right)^{-1/2}$$

The point where $\tau_{sw} = \tau_\omega$ does not correspond to the isotropization (Zeman) scales!